SECOND PUC MODEL QUESTION PAPER 2023-2024

SUBJECT : MATHEMATICS (35)

TIME : 3 Hours 15 Minutes [Total questions : 52] Max. Marks : 80

Instructions : 1. The question paper has five parts namely A, B, C, D and E. Answer all the Parts.

2. Part A has 15 multiple choice questions, 5 fill in the blank questions.

3. Use the graph sheet for question on linear programming problem in Part E.

PART -A

I. Answer all the multiple choice questions : 15 x 1 = 15

1. The relation R in the set { 1,2,3 } given by { (1,2) ,(2,1) } is
a) reflexive b) symmetric
c) transitive d) equivalence relation

2. If f : R → R be defined as f(x) = x^4,then the function f is
a) one-one and onto b) many-oneandonto
c) one-one but not onto d) neither one-one nor onto

3. The principal value branch of $\cot^{-1} x$ is
a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$
c) $[0,\pi]$ d) $(0,\pi)$

4. The number of all possible matrices of order 3 x 3 with each entry 0 or 1 is
a) 27 b) 18 c) 81 d) 512

5. Let A be a nonsigular matrix of order 3 x 3 and | adj A|= 25, thena possible value of |A| is
a) 625 b) 25 c) 5 d) 125

6. Which of the following x belongs to domain of the greatest integer function f(x) = [x] , 0 < x < 3 is not differentiable
a) 2 and 3 b) 1 and 2 c) 0 and 2 d) 1 and 3

7. If y = $\log_7 2x$, then $\frac{dy}{dx}$ is
1) $\frac{1}{x\ log 7}$ b) $\frac{1}{7\ logx}$
c) $\frac{logx}{7}$ d) $\frac{7}{logx}$

8. The point of inflection of the function y = x^3 is
a) (2, 8) b) (1, 1) c) (0, 0) d) (–3, -27)

9. $\int sin2x$ dx is
a) $-\frac{sin2x}{2}$ + c b) $-\frac{cos2x}{2}$ + c
c) $\frac{cos2x}{2}$ + c d) $\frac{sin2x}{2}$ + c

10. $\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)$ dx is
a) $e^{-x}\left(\frac{1}{x}\right)$ + c b) $e^{-x}\left(\frac{1}{x^2}\right)$ + c c) $e^x\left(\frac{1}{x}\right)$ + c d) $e^x\left(\frac{1}{x^2}\right)$ + c

11. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}\cdot\vec{b} = |\vec{a} \times \vec{b}|$, when $\tan\theta$ is equal to,

a) 1 b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) 0

12. Unit vector in the direction of $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ is

a) $\frac{2\hat{\imath}+3\hat{\jmath}+\hat{k}}{14}$ b) $\frac{2\hat{\imath}-3\hat{\jmath}+\hat{k}}{\sqrt{14}}$

c) $\frac{2\hat{\imath}+3\hat{\jmath}+\hat{k}}{\sqrt{14}}$ d) $\frac{2\hat{\imath}+3\hat{\jmath}-\hat{k}}{14}$

13. If the direction cosines l,m,n of a line are 0, $\frac{1}{2}$, $\frac{\sqrt{3}}{2}$ then the angle made by the line with the positive direction of y – axis is

a) 60^0 b) 30^0 c) 90^0 d) 45^0

14. In a Linear programming problem , the objective function is always

a) a cubic function b) a quadratic function

c) a linear function d) a constant function

15. If A and B are two non empty events such that $P(A/B) = P(B/A)$ and $P(A \cap B) \neq \emptyset$ then

a) $A \subset B$ but $A \neq B$ b) $A = B$

c) $B \subset A$ but $A \neq B$ d) $P(A) = P(B)$

II. Fill in the blanks by choosing the appropriate answer from those given in the bracket **5 x 1= 5**

$$\left(0,\quad 1,\quad 4,\quad \frac{1}{36},\quad 7,\quad \frac{1}{6}\right)$$

16. The value of $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$ is ______________

17. A square matrix A is a singular matrix if $|A|$ is ______________

18. The order of the differential equation $\frac{d^4y}{dx^4} + \sin(y^{III}) = 0$ is ______________

19. The lines $\frac{x-5}{k} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular, then k is ______________

20. The probability of obtaining an even prime number on each die, when a pair of dice is rolled is ______________

PART –B

Answer any six questions **6 x 2 =12**

21. Prove that $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

22. Find the equation of line joining (1, 2) , (3, 6) using determinant method

23. Find $\frac{dy}{dx}$, if $y + \sin y = \cos x$

24. Find the rate of change of the area of a circle with respect to its radius r when r = 3 cm

25. Find the local minimum value of the function f given by $f(x) = 3 + |x|$, $x \in R$

26. Find $\int \frac{dx}{(x+1)(x+2)}$

27. Evaluate $\int_0^{\frac{\pi}{2}} \left(\sin^2\frac{x}{2} - \cos^2\frac{x}{2}\right) dx$

28. Find the projection of the vector $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} + 2\hat{k}$ on the vector $\vec{b} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

29. Find the angle between the pair of lines given by
$\vec{r} = 3\,\hat{\imath} + 2\,\hat{\jmath} - 4\hat{k} + (\,\hat{\imath} + 2\,\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\,\hat{\imath} - 2\,\hat{\jmath} + \mu\,(3\hat{\imath} + 2\,\hat{\jmath} + 6\,\hat{k})$

30. A fair die is rolled. Consider events E = {1, 3, 5} , F = {2, 3} , find P (E/F)

31. If A and B two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \cap B) = \frac{1}{8}$,
find P (not A and not B)

PART – C

Answer any six questions **6 x 3 =18**

32. Show that the relation R in the set A = {1,2,3,4,5} given by
R = {(a, b): |a − b| is even } is an equivalence relation

33. Write in the simplest form $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$, $x \neq 0$

34. Express $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of a symmetric anda skew symmetric matrix.

35. Differentiate sin^2x with respect to e^{cosx}

36. Differentiate x^{sinx} , x > 0 with respect to x

37. Find the interval in which the function $f(x) = 10 - 6x - 2x^2$ is strictly increasing

38. Find $\int x \sin^{-1} x \; dx$

39. Find the equation of curve passing through the point (–2, 3) , given that the slope of the tangent to the curve at any point (x, y) is $\frac{2x}{y^2}$

40. Show that the position vector of the point P, which divides the line joining the points A and B having position vectors $\vec{a}$ and $\vec{b}$ internally in the ratio m: n is $\frac{m\,\vec{b} + n\vec{a}}{m+n}$

41. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = 3\,\hat{\imath} + 2\,\hat{\jmath} + 2\hat{k}$ and $\vec{b} = \hat{\imath} + 2\,\hat{\jmath} - 2\hat{k}$

42. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn at random from the bag and it is found to be red .Find the probability that the ballis drawn from first bag ?

PART – D

Answer any four questions **4 x 5 = 20**

43. Let f : N→ Y be a function defined as f(x) = 4x + 3, where Y = {y ∈ N : y = 4x + 3 for some x ∈ N}.Show that f is invertible. Find the inverse of f.

44. If $A = \begin{bmatrix} 0 & 6 & 7 \\ -6 & 0 & 8 \\ 7 & -8 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$ then calculateAC, BC and (A + B)C. Also verify (A + B) C = AC + B C

45. Solve the system of linear equations by matrix method
$2x - 3y + 5z = 11$, $3x + 2y - 4z = -5$, $x + y - 2z = -3$

46. If y = 3 cos(logx) + 4 sin (logx) , show that $x^2y_2 + xy_1 + y = 0$

47. Find the integral of $\frac{1}{x^2-a^2}$ with respect to x and hence evaluate $\int \frac{dx}{x^2-16}$

48. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ using integration.

49. Find the general solution of the differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$, $(x \neq 0)$

50. Derive the equation of a line in space through a given point and parallel to a vector both in the vector and Cartesian form

PART – E

Answer the following questions

51. P.T. $\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx\ , & \text{if } f(x) \text{is an even function} \\ 0 & \text{if } f(x) \text{is an odd function} \end{cases}$

and hence evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 x dx$ **6**

OR

Solve the following linear programming problem graphically
Minimise $Z = 200x + 500y$,
subject to the constraints : $x + 2y \geq 10$, $3x + 4y \leq 24$, $x \geq 0$, $y \geq 0$

52. Show that the matrix $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$,

where I is 2 x 2 identity matrix and O is 2 x 2 zero matrix.

Using this equation, find A^{-1} . **4**

OR

Find the value of k so that the function $f(x) = \begin{cases} \frac{k \cos x}{\pi - 2x} & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$

is continuous at $x = \frac{\pi}{2}$

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